The frequencies shown in Fig. 1 satisfy the first selection rule for nonlinear interactions,

$$f_1 \pm f_2 \pm f_3 = 0 \text{ and } f_2 \pm f_3 \pm f_4 = 0$$

If the fundamental frequency of the unstable shear layer is denoted as f_2, then note that the frequencies satisfy the rule

$$f_1 = f_3 - f_2 \text{ and } f_4 = f_3 + f_2$$

At this point, it is not clear whether the waves at frequencies f_1 an f_4 are nonlinearly produced from the waves at frequencies f_3 and f_2 or whether they are spontaneously excited independent waves associated with the acoustic characteristics of the wind tunnel. In order to distinguish between nonlinear coupled waves and linear independent waves, Kim et al.[10,11] have shown that digital implementation of a higher order spectrum known as the bispectrum is very useful in identifying nonlinear waves from a self-excited fluctuation spectrum.

Kim and Powers[10] have shown that digital implementation of the bispectrum provides a method for distinguishing between spontaneously excited and coupled modes by measuring the degree of phase coherence between modes. The bispectrum $B(\omega_1,\omega_2)$ corresponds to a two-dimensional Fourier transform of a second-order correlation function $C(\tau_1,\tau_2) = \langle x(t)x(t+\tau_1)x(t+\tau_2)\rangle$, where the bracket indicates the expectation value. Although the bispectrum is equivalent to a two-dimensional Fourier transform of $C(\tau_1,\tau_2)$ it may also be written as

$$B(\omega_1,\omega_2) = \lim_{T\to\infty} (1/T)\langle x(\omega_1)x(\omega_2)x^*(\omega_1+\omega_2)\rangle$$

where $x(\omega)$ is the Fourier transform of $x(t)$, T the time duration of the $x(t)$ signal, and the asterisk represents a complex conjugate.

The bispectrum will be zero unless waves are present at the frequencies ω_1, ω_2, and $\omega_1+\omega_2$ and, simultaneously, a phase coherence must be present between these waves. If the sum and difference frequency waves are generated through a nonlinear interaction, then a phase coherence exists. The statistical averaging will not lead to a zero value of the bispectrum. To carry out the demonstration for the first trio of frequencies shown in Fig. 2, measurements of the bispectral correlation coefficient were carried out. The bispectral correlation coefficient is defined as

$$\left\langle \frac{u'(f_1,t)}{\langle u'^2(f_1)\rangle^{1/2}} \cdot \frac{u'(f_2,t')}{\langle u'^2(f_2)\rangle^{1/2}} \cdot \frac{u'(f_3,t'')}{\langle u'^2(f_3)\rangle^{1/2}} \right\rangle$$

where $u'(t)$ is the velocity oscillation associated with each frequency and the angled brackets indicate the expectation value. To evaluate this parameter for the first three frequencies shown in Fig. 2, the Hewlett-Packard Model 2594A sweeping local oscillator was sequentially set at each of the first three frequencies. The HP Model 2240A measurement and control processor was used to take 256 digital data points, at the rate of 1000 samples/s of each of the wave forms from the band pass filters of the analyzer. Each resulting set of time-domain data was then fast-Fourier transformed into the frequency domain by the HP-85. These three sets of data were then used to compute the bispectral correlation coefficient. The resulting time-domain data were then transformed into the frequency domain which is presented in Fig. 3. The waveform analysis procedure developed by Hewlett-Packard Corporation for the HP-85 was used in these calculations.

The results presented in Fig. 3 indicate a high degree of correlation between the three frequencies, which is a requirement for quadratic interactions.

In addition to the frequency selection rule, the nonlinear terms of the Navier-Stokes equation introduce the wave number selection rule as[8,9]

$$k_1 \pm k_2 \pm k_3 = 0$$

where the wave number is defined as

$$k = 2\pi f / c_r(f)$$

where $c_r(f)$ is an appropriate wave phase velocity at that frequency. In order to show conclusively that these waves are of nonlinear interaction origin, it will be necessary to show that the wave number selection rule is also satisfied.

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A Fuzzy Algorithm to Compute Transonic Profile Flow

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Introduction

WHEN the governing equation of a system is nonlinear and no analytical solution is obtained, one has to depend on some iteration scheme. It is a general observation that the convergence of these schemes depends on the prechoice of the starting values. The convergence is ensured and the scheme will be faster if the starting solution more or less approximates the true solution. This approximation is,

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generally, made from a previous knowledge of the nature of the true solution. In this Note an attempt is made to obtain this approximate solution via a fuzzy algorithm. According to Zadeh,[1] "it is this fuzzy, and as yet not well-understood, logic that plays a basic role in what may well be one of the most important facets of human thinking, namely, the ability to summarize information—to extract from the collections of masses of data impinging upon the human brain those and only those subcollections which are relevant to the performance of the task at hand." In fact, a fuzzy algorithm is an ordered set of fuzzy instructions which upon execution yield an approximate solution to a specified problem. By a fuzzy instruction we mean an instruction of this nature: if M is not very large, approximate a nearly 1 or, if M is large, increase a slightly.

To describe the working principle, we consider here a typical problem i.e., steady plane inviscid supercritical transonic flow past thin profile at zero incidence with shock.

Formulation of the Problem

The gasdynamic equation for two-dimensional transonic profile flow at zero incidence may be converted into a two-dimensional, singular, nonlinear integral equation, known as the integral equation of Oswatitsch. An approximate solution of that equation[2] is

$$U(X,Y)=\ell(a)\left[1\pm\left\{1-\frac{2}{\ell(a)}\left(U_p(X,Y)-I(X,Y)\right)\right\}^{1/2}\right] \quad (1)$$

where

$$I(X,Y)=\frac{1}{\pi}\int_{-\infty}^{\infty}\!\!\int\frac{(2a-1)(\xi-X)^2-(2a+1)(\eta-Y)^2}{[(2a-1)(\xi-X)^2+(2a+1)(\eta-Y)^2]^2}$$

$$\times[2U(\xi,\eta)-2U_p(\xi,\eta)+(a-1/2)U^2(\xi,\eta)]\mathrm{d}\xi\mathrm{d}\eta \quad (2a)$$

and

$$\ell(a)=1+\frac{\sqrt{4a^2-1}}{2a-1} \quad (2b)$$

It is evident from Eq. (2b) that $a>1/2$ and $\ell(a)\to 2$ as $a\to\infty$. For shock-free cases only the negative sign in Eq. (1) was considered. It can be shown that for shock-free cases the value of the integral $I(X,Y)$ is negligible. Taking $a=1$ and considering the negative sign in Eq. (1), results were computed for a number of profiles.[3] Satisfactory agreement with the exact solutions was obtained. But it is not as easy to obtain a solution with shock using Eq. (1); with the negative sign Eq. (1) gives a shock-free continuous solution; with the positive sign it gives purely supersonic flow. Hence we have tried to attack the problem by properly using both signs.

Fuzzy Algorithm

Since solution (1) with a positive sign gives purely supersonic flow, and $I(X,Y)$ is small, we have used

$$U(X,Y)=\ell(a)\left[1+\left\{1-\frac{2}{\ell(a)}U_p(X,Y)\right\}^{1/2}\right] \quad (3)$$

in the region from the point where $U_p(X,Y)$ is maximum, up to the decelerating sonic point, and

$$U(X,Y)=\ell(a)\left[1-\left\{1-\frac{2}{\ell(a)}U_p(X,Y)\right\}^{1/2}\right] \quad (4)$$

otherwise. There remains a problem of choosing $\ell(a)$. This is to be chosen such that the flow would be smooth up to the discontinuity at the decelerating sonic point, and $\ell(a)$ must be greater than 2. This is achieved by applying a fuzzy decisional algorithm.[1] We call it a fuzzy algorithm of type I. Actually it is a fuzzy algorithm which serves to provide an approximate description of a decisional rule.

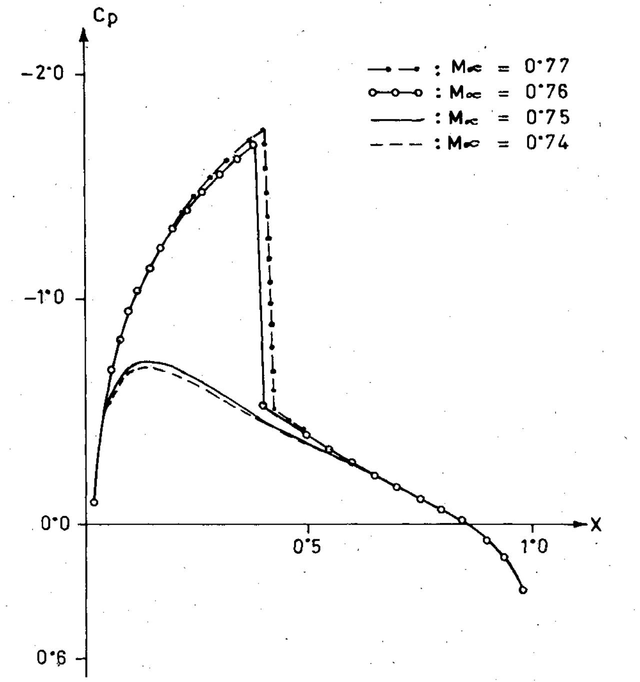

Fig. 1 Pressure distribution curves for different Mach numbers for NACA 0012 using a type I fuzzy algorithm.

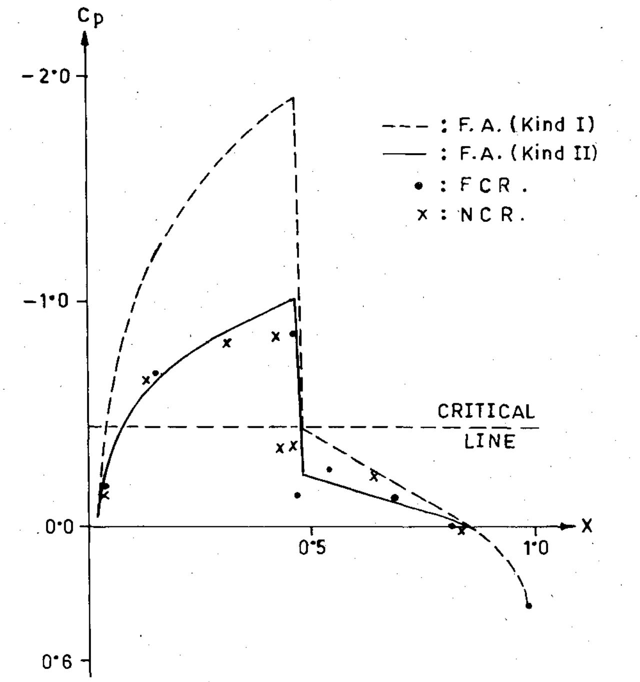

Fig. 2 Fuzzy estimation of pressure distribution for NACA 0012 profile ($M_\infty=0.791$).

Fuzzy Algorithm (Type I)

This determines whether the solution is shock-free or with shock, and if with shock then the pattern of the solution. To construct this algorithm one should always keep in mind that the solution is continuous up to the compression shock. For each prescribed profile shape and given freestream Mach number, we have a nonfuzzy set of U for different $\ell(a)$. A fuzzy set is generated by assigning to each element of the nonfuzzy set of U a fuzzy measure. This measure is its similarity with the known patterns of transonic flow with shock at zero incidence. We select the element whose fuzzy measure is maximum. In fact, this corresponds to $\ell(a)=2(U_p)_{max}$. Since $\ell(a)\geq 2$, a discontinuous solution

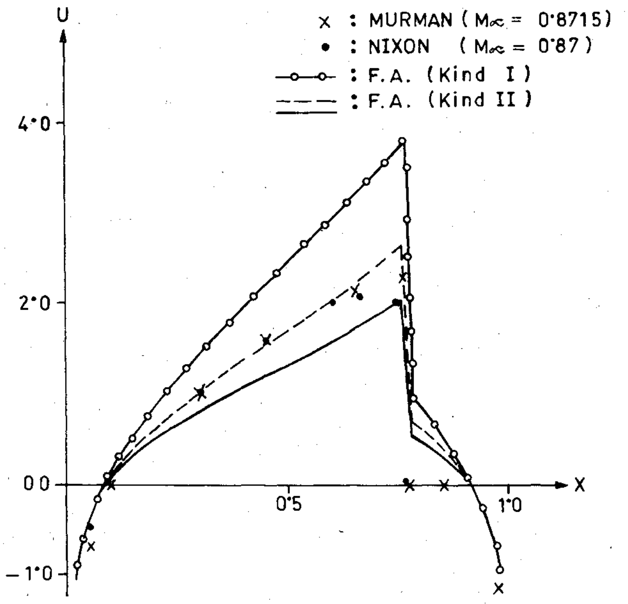

Fig. 3 Fuzzy estimation of velocity distribution for a parabolic arc profile.

similar to that with shock would be obtained only if $(U_p)_{\max} \geq 1$. In Fig. 1, results for NACA 0012 profile for different freestream Mach numbers have been plotted. A continuous solution has been obtained up to $M_\infty = 0.75$. After that a discontinuous solution appears. Agreement with the shock position is excellent, as is shown in Figs. 2 and 3. Disagreement is mainly in shock strength in both the cases.

Fuzzy Algorithm (Type II)

For the modification of the result obtained by type I, a fuzzy generation algorithm,[1] or fuzzy algorithm of type II, is applied. This algorithm serves to generate different consistent patterns. Following this algorithm a set of feasible patterns will be obtained and then from these feasible patterns, the most preferable one will be selected.[4] But, as we shall see in the following discussion, it is better to choose a set of preferable patterns than the conventional notion of choosing the most preferable one.[5]

It was observed in Ref. 2 that the optimal value of a in Eq. (2b) is 1 for shock-free supercritical flow. But for increasing Mach number and for flow with shock, the value of $U(X,Y)$ at the point of $(U_p)_{\max}$ is found to be a good fit for decreasing a in Eq. (2b). Moreover, it appears from Eq. (2) that $I(X,Y)$ will be smaller for the value of a nearly ½. Thus, our choice of a is

$$a = \tfrac{1}{2} + \epsilon \qquad (5)$$

where ϵ is a small positive real number. Different scale factors could be evaluated using different ϵ corresponding to the point of $(U_p)_{\max}$ on the profile axis. A set of results would be generated using these scale factors on the discontinuous solution obtained using type I. In Fig. 3, results have been shown for parabolic arc profile for $M_\infty = 0.87$. The continuous line is for $\epsilon = 0.01$; the dotted line is for $\epsilon = 0.31$. The set of preferable patterns could be defined for ϵ in the interval [0.01, 0.31]. A more restricted choice of ϵ could be possible, at the user's discretion. In the present work ϵ is defined as:

$$\epsilon = \frac{M_\infty}{[(U_p)_{\max}]^2} \qquad (6)$$

For the particular case in Fig. 3 it becomes 0.1, which is in the defined interval. This definition (6) is used for the NACA 0012 profile for $M_\infty = 0.791$, and results are shown in Fig. 2. The overall agreement is quite satisfactory.

The novel feature of these fuzzy algorithms is the huge time reduction. CPU time for the computation of the result shown in Fig. 2 or 3 is approximately 3 s on the B 6700. We are now trying to extend this method to three-dimensional cases and to generalize this approach for various applications.

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Axial Wavenumber Measurements in Axisymmetric Jets

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Introduction

THE existence of orderly structures in axisymmetric jets of air has been well established since the first observations by Mollo-Christensen.[1] Since that time, many investigators[2-12] have studied the coherent structures and their importance in the generation of noise.

Kibens[7] showed that, for incompressible flows ($M<0.3$) with a laminar shear layer at the exit of the jet, an instability was generated whose wavelength scaled on the thickness of the shear layer. As the flow progressed downstream, the shear layer became thicker and the frequency of the coherent structure halved according to a vortex pairing phenomenon, such that the wavelength of the new spectral component was again scaled upon the thickness of the shear layer. Eventually, after several pairings during the first few diameters of the flow, a coherent structure whose wavelength scaled on the diameter of the jet was obtained and the vortex pairing ceased. The resulting Strouhal number (fD/Uo) was around 0.4.

This behavior is very different from the compressible jets ($M \geq 1.4$) studied by Morrison and McLaughlin.[9] The supersonic jets studied possessed laminar shear layers at the exit of the nozzle. However, the initial instability observed in the compressible jets had wavelengths which scaled upon the diameter of the jet. In addition, there was no subharmonic production in the supersonic jets and, hence, no direct evidence of vortex pairing.

Kibens also observed that, if the shear layer was initially turbulent in the incompressible jet, then the pairing process did not appear to occur. The jet initially developed a coherent

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